



WORKSHEET

Statistics 1

Geometric Distribution Question

- Notation : $\text{Geo}(p)$ $P(\text{Success}) = p$
 $P(\text{failure}) = 1 - p$
 $X = \#$ of trials for 1st success to occur.
- Probability density function for Geo distribution : For 1st success to occur on x trial
 - ① ' $x-1$ ' trials must fail. (probability = $(1-p)^{x-1}$)
 - ② ' x ' trial must be a success. (probability = p)

$$\therefore \boxed{P(X=x) = (1-p)^{x-1} p}$$

$$\boxed{\mu = E(X) = \frac{1}{p}}$$

QUESTIONS

Q1 s2020_51

The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

- (a) Show that the probability that the score is 4 is $\frac{1}{12}$. [1]

The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X .

- (b) Find the mean of X . [1]

- (c) Find the probability that a score of 4 is first obtained on the 6th throw. [1]

- (d) Find $P(X < 8)$. [2]

Q2 s2020_53

A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

- (a) Find the expected value of X . [1]

- (b) Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

- (c) Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

Q3 w2020_51

Kayla is competing in a throwing event. A throw is counted as a success if the distance achieved is greater than 30 metres. The probability that Kayla will achieve a success on any throw is 0.25.

- (a) Find the probability that Kayla takes more than 6 throws to achieve a success. [2]

- (b) Find the probability that, for a random sample of 10 throws, Kayla achieves at least 3 successes. [3]

Q4 w2020_52

A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

- (a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

On another occasion, the die is thrown 10 times.

- (b) Find the probability that a 4 is obtained at least 3 times. [3]

Q5 w2020_53

An ordinary fair die is thrown until a 6 is obtained.

- (a) Find the probability that obtaining a 6 takes more than 8 throws. [2]

Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X .

- (b) Find the expected value of X . [1]

- (c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws. [2]

Q6 s2021_52

An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X .

- (a) Write down the mean of X . [1]

- (b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw. [2]

- (c) Find the probability that a 5 is first obtained in fewer than 10 throws. [2]

Q7 s2021_53

Three fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.

- (a) Find the probability that the score is 4 on a single throw of the three dice. [3]
- (b) Find the probability that a score of 18 is obtained for the first time on the 5th throw of the three dice. [3]

Q8 w2021_51

Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time.

- (a) Find the probability that two tails are obtained for the first time on the 7th throw. [2]
- (b) Find the probability that it takes more than 9 throws to obtain two tails for the first time. [2]

Q9 w2021_52

In a certain region, the probability that any given day in October is wet is 0.16, independently of other days.

- (a) Find the probability that, in a 10-day period in October, fewer than 3 days will be wet. [3]
- (b) Find the probability that the first wet day in October is 8 October. [2]
- (c) For 4 randomly chosen years, find the probability that in exactly 1 of these years the first wet day in October is 8 October. [2]

Q10 s2022_53

Ramesh throws an ordinary fair 6-sided die.

- (a) Find the probability that he obtains a 4 for the first time on his 8th throw. [1]
- (b) Find the probability that it takes no more than 5 throws for Ramesh to obtain a 4. [2]

ANSWERS

① $\text{Score} = d_1 + d_2$

(a)
$$\begin{aligned} P(\text{Score} = 4) &= P(1,3) + P(3,1) + P(2,2) \\ &= \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) \\ &= \frac{3}{36} = \frac{1}{12} \end{aligned}$$

$$\therefore \boxed{P(\text{Score} = 4) = \frac{1}{12}}$$

(b) $X = \#$ of trials till score 4 obtained.

$$P(\text{success}) = p = \frac{1}{12}$$

$$\therefore \mu = E(X) = \frac{1}{p} = \frac{1}{\left(\frac{1}{12}\right)} = 12$$

$$\therefore \boxed{\mu = 12}$$

(Takes 12 rolls on average to obtain a score 4)

(c) $P(X=6) = ?$ As $P(X=x) = (1-p)^{x-1} p$

$$\therefore P(X=6) = \left(1 - \left(\frac{1}{12}\right)\right)^{6-1} \left(\frac{1}{12}\right) = \left(\frac{11}{12}\right)^5 \left(\frac{1}{12}\right)$$

$$\therefore \boxed{P(X=6) = 0.0539}$$

(d) $P(X < 8) = ?$ $P(X \geq 8) =$ probability it fails 7 times in a row.

$$\therefore P(X \geq 8) = \left(1 - \frac{1}{12}\right)^7 = \left(\frac{11}{12}\right)^7 \quad (\text{Score 4 isn't obtained 7 times in a row})$$

$$\therefore P(X < 8) = 1 - P(X \geq 8) = 1 - \left(\frac{11}{12}\right)^7$$

$$\therefore \boxed{P(X < 8) = 0.456}$$

② $X = \#$ of trials to obtain 2 tails.

(a) $p = \text{probability of obtaining 2 tails}$
 $p = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

$$\therefore E(X) = \frac{1}{\left(\frac{1}{4}\right)} = 4$$

$$\boxed{\mu = 4}$$

(b) $P(X=3) = \left(1 - \left(\frac{1}{4}\right)\right)^{3-1} \left(\frac{1}{4}\right)$
 $= \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)$

$$\therefore \boxed{P(X=3) = 0.141}$$

(c) $P(X < 6) = 1 - P(X \geq 6)$

$P(X \geq 6) = \text{Probability of 5 failures in a row}$

$$\therefore P(X \geq 6) = \left(\frac{3}{4}\right)^5$$

$$\therefore P(X < 6) = 1 - \left(\frac{3}{4}\right)^5$$

$$\therefore \boxed{P(X < 6) = 0.763}$$

$$(3) \quad p = 0.25$$

(a) $X = \#$ of throws to achieve a success

$P(X > 6) =$ Probability of 6 failures in a row.

$$\therefore P(X > 6) = (0.75)^6$$

$$\therefore \underline{P(X > 6) = 0.178}$$

(b) $n = 10$ $X = \#$ of successes in 10 trials

$$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - (P(X=0) + P(X=1) + P(X=2))$$

$$= 1 - \left(\binom{10}{0} (0.25)^0 (0.75)^{10} + \binom{10}{1} (0.25)^1 (0.75)^9 + \binom{10}{2} (0.25)^2 (0.75)^8 \right)$$

$$\therefore \underline{P(X \geq 3) = 0.474}$$

(4) $X = \#$ of trials to obtain 4.

(a) $p = \frac{1}{6}$ $P(X < 6) = 1 - P(X \geq 6)$

$$P(X \geq 6) = \text{Prob of 5 failures in a row} \\ = \left(\frac{5}{6}\right)^5$$

$$\therefore P(X < 6) = 1 - \left(\frac{5}{6}\right)^5 = 0.598$$

$$\therefore \boxed{P(X < 6) = 0.598}$$

(b) $n = 10$ $X = \#$ of successes in 10 trials.

$$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - (P(X=0) + P(X=1) + P(X=2))$$

$$= 1 - \left(\binom{10}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} + \right. \\ \left. \binom{10}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 + \right. \\ \left. \binom{10}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^8 \right)$$

$$\therefore \boxed{P(X \geq 3) = 0.225}$$

(5) $X = \#$ of trials until 6 obtained.

(a) $P(X > 8) = \text{Prob of 8 failures in a row}$
 $p = \frac{1}{6}$

$$\therefore P(X > 8) = \left(\frac{5}{6}\right)^8 = \underline{\underline{0.233}}$$

(b) $X = \#$ of trials to obtain 2 6's.

$$p = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$$

$$E(X) = \frac{1}{\left(\frac{1}{36}\right)} = \underline{\underline{36}}$$

$$\begin{aligned} (c) \quad P(X=10, 11) &= P(X=10) + P(X=11) \\ &= \left(\frac{35}{36}\right)^9 \left(\frac{1}{36}\right) + \left(\frac{35}{36}\right)^{10} \left(\frac{1}{36}\right) \end{aligned}$$

$$\boxed{P(X=10, 11) = 0.0425}$$

(6) $X = \#$ of trials to obtain 5.
 $p = 1/6$

(a) $E(X) = \frac{1}{\frac{1}{6}} = \underline{\underline{6}}$

(b) $P(3 < X < 8) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$
 $= \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6 \left(\frac{1}{6}\right)$

$\therefore \boxed{P(3 < X < 8) = 0.2996}$

(c) $P(X < 10) = 1 - \underbrace{P(X \geq 10)}_{9 \text{ fails in a row}}$

$\therefore P(X < 10) = 1 - \left(\frac{5}{6}\right)^9 = 0.806$

$\boxed{P(X < 10) = 0.806}$

$$\begin{aligned} \textcircled{7} (a) \quad P(\text{Score} = 4) &= P(1, 1, 2) + P(1, 2, 1) + P(2, 1, 1) \\ &= \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^3 \\ &= 3\left(\frac{1}{6}\right)^3 = \frac{3}{216} = \frac{1}{72} \end{aligned}$$

$$\boxed{P(\text{Score} = 4) = \frac{1}{72}}$$

$$(b) \quad p = P(\text{Score} = 18) = P(6, 6, 6) = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$$

$X = \#$ of trials to obtain score 18

$$\therefore P(X = 5) = \left(\frac{215}{216}\right)^4 \left(\frac{1}{216}\right)$$

$$\therefore \underline{\underline{P(X=5) = 0.0454}}$$

(8) $X = \#$ of trials to obtain 2 tails.

(a) $p = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$$P(X=7) = \left(\frac{3}{4}\right)^6 \left(\frac{1}{4}\right) = 0.0445$$

$$\therefore \boxed{P(X=7) = 0.0445}$$

(b) $\underbrace{P(X>9)}_{\substack{\text{9 fails in a} \\ \text{row}}} = \left(\frac{3}{4}\right)^9$

$$\therefore \boxed{P(X>9) = 0.0751}$$

(9) (a) $p = 0.16$ $X = \#$ of days in October that are wet.

$$\begin{aligned} P(X < 3) &= P(X=0) + P(X=1) + P(X=2) \\ &= \binom{10}{0} (0.16)^0 (0.84)^{10} + \\ &\quad \binom{10}{1} (0.16)^1 (0.84)^9 + \\ &\quad \binom{10}{2} (0.16)^2 (0.84)^8 \end{aligned}$$

$$\therefore \boxed{P(X < 3) = 0.7914}$$

(b) $X = \#$ of trials to get wet day

$$P(X=8) = (0.84)^7 (0.16)$$

$$\therefore \boxed{P(X=8) = 0.0472}$$

(c) $p = 0.0472$ $n = 4$

$X = \#$ of years where the first wet day in October is the 8th

$$P(X=1) = \binom{4}{1} (0.0472)^1 (0.9528)^3$$

$$\therefore \boxed{P(X=1) = 0.163}$$

(10) $X = \#$ of trials to obtain a 4.

$$(a) \quad P(X=8) = \left(\frac{5}{6}\right)^7 \left(\frac{1}{6}\right)$$

$$\therefore \boxed{P(X=8) = 0.0465}$$

$$(b) \quad P(X \leq 5) = 1 - \underbrace{P(X \geq 5)}_{\substack{5 \text{ failures} \\ \text{in a row}}} = 1 - \left(\frac{5}{6}\right)^5$$

$$\therefore \boxed{P(X \leq 5) = 0.598}$$